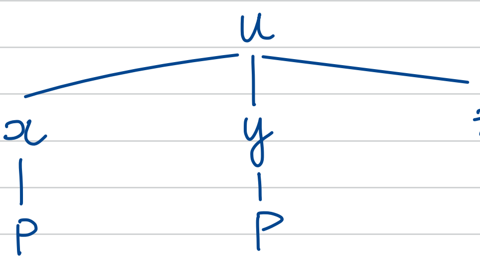
Q1: If $u=2x^3y^2-z^3$, where $x=2p-3p^2$, $y=pe^{2p}$, and $z=p\cos(p)$, use the Chain Rule to find $\dfrac{du}{dp}$.



$$\frac{du}{dp} = \frac{\partial u}{\partial x}\frac{dx}{dp} + \frac{\partial u}{\partial y}\frac{dy}{dp} + \frac{\partial u}{\partial z}\frac{dz}{\partial p}$$

$$= (6x^2y^2)(2-6p) + (4x^3y)(e^{2p}(1+2p)) + (-3z^2)(\cos p - p\sin p)$$

where $x = 2p-3p^2$ $\quad y = pe^{2p}$ $\quad z = p\cos p$

Q2: If $v=x^2\cos(y)+ye^{xy}$, where $x=s+2t$ and $y=st$, use the Chain Rule to find $\dfrac{\partial v}{\partial s}$ and $\dfrac{\partial v}{\partial t}$ when $s=0$ and $t=0$.

1) $$\frac{\partial v}{\partial s} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial s}$$

$$= (2x\cos y + y^2e^{xy})(1) + (-x^2\sin y + e^{xy}(1+xy))(t)$$

At $s=0, t=0$: $x = 0+2(0) = 0$ $\quad y = 0(0) = 0$

$$\left.\frac{\partial v}{\partial s}\right|_{s=0,\, t=0} = (2\cdot 0\cdot\cos 0 + 0^2e^0) + (-0^2\sin 0 + e^0(1+0))(0)$$

$$= 0$$

2) $\frac{\partial v}{\partial t} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial t}$

$= (2x\cos y + y^2 e^{xy})(2) + (-x^2 \sin y + e^{xy}(1+xy))(s)$

$= (2 \cdot 0 \cdot \cos 0 + 0^2 e^0)(2) + (-0^2 \sin 0 + e^0(1+0))(0)$

$= 0$

Q3: If $z = y + f(x^2 - y^2)$, where f is differentiable, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$.

① Let $u = x^2 - y^2 \Rightarrow z = y + f(u)$

② $\frac{\partial z}{\partial x} = \frac{d}{dx}(y + f(x^2 - y^2)) = 0 + f'(u)\frac{\partial}{\partial x}(x^2 - y^2) = f'(u) \cdot 2x$

$= 2x f'(x^2 - y^2)$

$\frac{\partial z}{\partial y} = \frac{d}{dy}(y + f(x^2 - y^2)) = 1 + f'(u)\frac{\partial}{\partial y}(x^2 - y^2) = 1 + f'(u) \cdot (-2y)$

$= 1 - 2y f'(x^2 - y^2)$

③ $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = y(2x f'(x^2 - y^2)) + x(1 - 2y f'(x^2 - y^2))$

$= 2xy f'(x^2 - y^2) + x - 2xy f'(x^2 - y^2)$

$= x$

QUICK TIME!

(a) When is the directional derivative of f a maximum?
(b) When is it a minimum?
(c) When is it 0?
(d) When is it half of its maximum value?

a) $D_{\vec{u}} f = \nabla f \cdot \vec{u} = \|\nabla f\| \, \|\vec{u}\| \cos\theta = \|\nabla f\| \cos\theta$

max when $\cos\theta = 1 \Rightarrow \theta = 0$
ie. $\vec{u}$ is same direction as ∇f

b) min when $\cos\theta = -1 \Rightarrow \theta = \pi$
ie. $\vec{u}$ is opposite direction of ∇f

c) $D_{\vec{u}} f = 0 \iff \cos\theta = 0$
ie. $\vec{u}$ is orthogonal to ∇f

d) $D_{\vec{u}} f = \underbrace{\frac{1}{2}\|\nabla f\| = \|\nabla f\|\cos\theta}_{\text{half of max}} \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

Q5: Find the directional derivative of f at the given point in the indicated direction.

$f(x,y) = x^2 e^{-y}$, $(-2,0)$
in the direction toward the point $(2,-3)$

① $\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \langle 2xe^{-y}, -x^2 e^{-y} \rangle$

② At $(-2,0)$: $\nabla f = \langle -2\cdot 2\cdot e^{0}, -(-2)^2 e^{-0} \rangle = \langle -4, -4 \rangle$

③ Unit direction vector from $(-2,0)$ to $(2,-3)$

$\vec{v} = \langle 2-(-2), -3-0 \rangle = \langle 4, -3 \rangle$

$\|\vec{v}\| = \sqrt{4^2 + (-3)^2} = 5$

$\vec{u} = \langle \frac{4}{5}, -\frac{3}{5} \rangle$

④ $D_u f = \nabla f \cdot \vec{u} = \langle -4, 4 \rangle \cdot \langle \frac{4}{5}, \frac{-3}{5} \rangle = -\frac{4^2}{5} - \frac{4(-3)}{5} = -4/5$

Q6: Find the direction in which $f(x,y,z) = ze^{xy}$ increases most rapidly at the point $(0,1,2)$. What is the maximum rate of increase?

$\nabla f = \langle zye^{xy}, zxe^{xy}, e^{xy} \rangle$

Direction of most rapid increase

$\nabla f|_{(0,1,2)} = \langle 2, 0, 1 \rangle$

Max. rate of increase

$\|\nabla f|_{(0,1,2)}\| = \sqrt{2^2+0^2+1^2} = \sqrt{5}$