Q1: Let $\bar{r}(t)=\left\langle \sqrt{2-t}, \dfrac{e^t-1}{t}, \ln(t+1)^3 \right\rangle$.

(a) Find the domain of $\bar{r}(t)$.

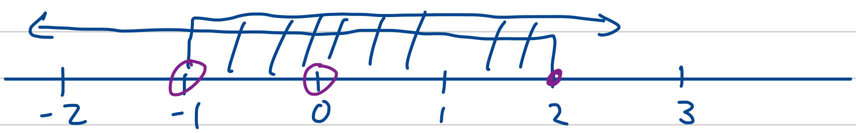
(b) Find $\lim\limits_{t\to 0}\bar{r}(t)$.

(c) Find $\bar{r}'(t)$.

a) $\sqrt{2-t}$: $2-t \geq 0 \Rightarrow t \leq 2$

$\dfrac{e^t-1}{t}$: $t \neq 0$

$\ln(t+1)^3$: $t+1>0 \Rightarrow t>-1$



$(-1,0) \cup (0,2]$

b) $\lim\limits_{t\to 0}\sqrt{2-t}=\sqrt{2}$ $\quad \lim\limits_{t\to 0}\dfrac{e^t-1}{t} \overset{LH}{=} \lim\limits_{t\to 0}\dfrac{e^t}{1}=1$

$\lim\limits_{t\to 0}\ln(t+1)^3=0$

$\therefore \lim\limits_{t\to 0}\vec{r}(t)=\langle\sqrt{2},1,0\rangle$

c) $\dfrac{d}{dt}\vec{r}(t)=\left\langle \dfrac{d}{dt}\sqrt{2-t}, \dfrac{d}{dt}\dfrac{e^t-1}{t}, \dfrac{d}{dt}\ln(t+1)^3 \right\rangle$

$=\left\langle \dfrac{-1}{2\sqrt{2-t}}, \dfrac{te^t-e^t+1}{t^2}, \dfrac{3}{t+1} \right\rangle$

Q2: Find parametric equations for the tangent line to the curve

$x=2\sin(t)$, $y=2\sin(2t)$, $z=2\sin(3t)$ at the point $(1,\sqrt{3},2)$

① Find t corresponding to $P(\overset{x}{1},\overset{y}{\sqrt{3}},\overset{z}{2})$

$x=2\sin(t) \Rightarrow \sin(t)=1/2 \Rightarrow t=\pi/6$

check: $2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$, $2\sin\left(\frac{3\pi}{6}\right) = 2$

② Get directional vector

$$x' = \frac{d}{dt} 2\sin t \quad \Rightarrow \quad 2\cos t$$

$$y' = \frac{d}{dt} 2\sin 2t \quad \Rightarrow \quad 4\cos 2t$$

$$z' = \frac{d}{dt} 2\sin 3t \quad \Rightarrow \quad 6\cos 3t$$

At $t = \pi/6$, $x'\left(\frac{\pi}{6}\right) = 2\cos \pi/6 = \sqrt{3}$

$$y'\left(\frac{\pi}{6}\right) = 4\cos\left(\frac{2\pi}{6}\right) = 2$$

$$z'\left(\frac{\pi}{6}\right) = 6\cos\left(\frac{3\pi}{6}\right) = 0$$

③ Eqⁿ: $\begin{cases} x = 1 + \sqrt{3}s \\ y = \sqrt{3} + 2s \\ z = 2 \end{cases}$

Q5: Find the length of the curve $\bar{r}(t) = \left\langle 2t^{3/2}, \cos(2t), \sin(2t) \right\rangle$ for $0 \le t \le 1$.

$$\bar{r}'(t) = \langle 3t^{1/2}, -2\sin(2t), 2\cos(2t) \rangle$$

$$\|\bar{r}'(t)\| = \sqrt{(3t^{1/2})^2 + (-2\sin 2t)^2 + (2\cos 2t)^2}$$

$$= \sqrt{9t + 4\sin^2(2t) + 4\cos^2(2t)}$$

$$= \sqrt{9t + 4}$$

$$L = \int_0^1 \sqrt{9t+4}\, dt = \int_4^{13} \sqrt{u}\, \frac{1}{9} du = \frac{1}{9}\left(\frac{2}{3} u^{\frac{3}{2}}\right)\Big|_4^{13}$$

$u = 9t + 4$ $\quad u(0) = 4 \quad u(1) = 13$

$du = 9dt$

$$= \frac{2}{27}\left(13^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$

Q6: For the given curve $\bar{r}(t)=\left\langle \sin^3(t),\cos^3(t),\sin^2(t)\right\rangle$ for $0\le t\le \pi/2$, find the unit tangent vector.

$\vec{r}'(t) = \langle 3\sin^2 t\cos t, -3\cos^2 t\sin t, 2\sin t\cos t\rangle$

$\|\vec{r}'(t)\|^2 = (3\sin^2 t\cos t)^2 + (-3\cos^2 t\sin t)^2 + (2\sin t\cos t)^2$

$= 9\sin^4 t\cos^2 t + 9\cos^4 t\sin^2 t + 4\sin^2 t\cos^2 t$

$= \sin^2 t\cos^2 t\,(9\sin^2 t + 9\cos^2 t + 4)$

$= \sin^2 t\cos^2 t\,(9+4) = 13\sin^2 t\cos^2 t$

$\|r'(t)\| = \sqrt{13}\sin t\cos t$

$\vec{T}_u(t) = \dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$

$= \dfrac{1}{\sqrt{13}\sin t\cancel{\cos t}}\langle 3\sin^2 t\cancel{\cos t}, -3\cos^2 t\cancel{\sin t}, 2\sin t\cancel{\cos t}\rangle$

$= \left\langle \dfrac{3\sin t}{\sqrt{13}}, \dfrac{-3\cos t}{\sqrt{13}}, \dfrac{2}{\sqrt{13}}\right\rangle$