Q1: Find two unit vectors that are orthogonal to both $\bar{j}+2\bar{k}$ and $\bar{i}-2\bar{j}+3\bar{k}$.

$\vec{u} = \langle 0,1,2 \rangle \qquad \vec{v} = \langle 1,-2,3 \rangle$

① Cross Product $\quad \vec{n} = \dfrac{\vec{u}\times\vec{v}}{\|\vec{u}\times\vec{v}\|} = \dfrac{1}{3\sqrt{6}}\langle 7,2,-1 \rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & -2 & 3 \end{vmatrix} = \vec{i}\left(1\cdot 3-2(-2)\right) - \vec{j}\left(0\cdot 3-2\cdot 1\right) + \vec{k}\left(0\cdot -2-1(1)\right)$$

$$= \vec{i}(3+4) - \vec{j}(-2) + \vec{k}(-1)$$

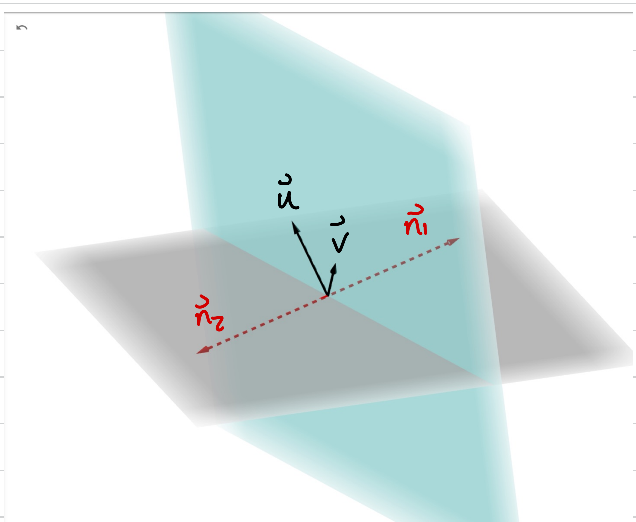
$$= \langle 7,2,-1 \rangle$$

② Normalize

$$\|\vec{u}\times\vec{v}\| = \sqrt{(7)^2+(2)^2+(-1)^2} = \sqrt{54} = 3\sqrt{6}$$

③ The unit vector orthogonal to both $\vec{u}$ and $\vec{v}$ are

$$\vec{n}_1 = \frac{1}{3\sqrt{6}}\langle 7,2,-1 \rangle$$



Q3: Determine if the statement is true or false. Justify your answers.

(a) For any vectors $\bar{u}$ and $\bar{v}$ in V^3, $|\bar{u}\cdot\bar{v}| = |\bar{u}||\bar{v}|$

(b) For any vectors $\bar{u}$ and $\bar{v}$ in V^3, $|\bar{u}\times\bar{v}| = |\bar{v}\times\bar{u}|$

(c) For any vectors $\bar{u}$ and $\bar{v}$ in V^3, $(\bar{u}\times\bar{v})\cdot\bar{u} = 0$

(d) If $\bar{u}\times\bar{v} = \bar{0}$, then $\bar{u} = \bar{0}$ or $\bar{v} = \bar{0}$

a) FALSE

$|\vec{u}\cdot\vec{v}| = |\vec{u}||\vec{v}||\cos\theta|$ in general.

Since $|\cos\theta| \le 1$, $|\vec{u}\cdot\vec{v}| = |\vec{u}||\vec{v}||\cos\theta| \le |\vec{u}||\vec{v}|$ only when

$\theta = 0$ or π

b) TRUE

Since $\vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$, vectors are opposite with magnitudes same.

c) TRUE

$\vec{u}\times\vec{v} = \vec{n} \perp$ to both $\vec{u}$ and $\vec{v}$.

So: $(\vec{u}\times\vec{v})\cdot\vec{u} = (\vec{u}\times\vec{v})\cdot\vec{v} = 0$

d) FALSE

$\vec{u}\times\vec{v} = \vec{0} \iff \vec{u} \parallel \vec{v}$ <u>or</u> $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$

Take $\vec{u} = \langle 1,2,3\rangle$ $v = 2\vec{u} = \langle 2,4,6\rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix} = i(12-12) - j(6-6) + k(4-4) = \vec{0}$$

Q5: Determine whether the two lines: intersect at a point, parallel, or skew.

(a) Line 1: $x=2+t$, $y=2+3t$, $z=3+t$
Line 2: $x=2+t$, $y=3+4t$, $z=4+2t$

(b) Line 1: $x=1+7t$, $y=3+t$, $z=5-3t$
Line 2: $x=4-t$, $y=6$, $z=7+2t$

(c) Line 1: $x=3-2t$, $y=4+t$, $z=6-t$
Line 2: $x=5-4t$, $y=-2+2t$, $z=7-2t$

a) $\vec{l}_1(t) = \langle 2+t, 2+3t, 3+t \rangle$ $\quad$ $\vec{l}_2(t) = \langle 2+s, 3+4s, 4+2s \rangle$

$\vec{d}_1 = \langle 1,3,1 \rangle$ $\quad$ $\vec{d}_2 = \langle 1,4,2 \rangle$

× Parallel — $\vec{d}_1 \neq k\vec{d}_2$ ie. not scalar multiples

× Intersect — $\left.\begin{array}{l} 2+t = 2+s \\ 2+3t = 3+4s \\ 3+t = 4+2s \end{array}\right\}$ $\begin{array}{l} t = s \\ 3t = 1+4s \leftarrow \text{bad} \end{array}$

∴ Skew

b) $\vec{d}_1 = \langle 7,1,-3 \rangle$ $\quad$ $\vec{d}_2 = \langle -1,0,2 \rangle$

× Parallel

Intersect — $\begin{array}{l} 1+7t = 4-s \\ 3+t = 6 \end{array}$ $\begin{array}{l} \rightarrow s+7t = 3 \\ \rightarrow t = 3 \end{array}\Big\}$ $s = -18$

$5-3t = 7+2s$

$\rightarrow 5-3(3) \stackrel{?}{=} 7+2(-18)$

$-4 \neq -29$

∴ Skew

c) $\vec{d}_1 = \langle -2, 1, -1 \rangle$ $\quad \vec{d}_2 = \langle -4, 2, -2 \rangle$

$2\vec{d}_1 = \vec{d}_2 \quad \therefore \quad \vec{l}_1 \parallel \vec{l}_2$

Q7: Determine whether the lines and the plane are parallel, perpendicular, or neither.

(a) line: $x = 3 - t$, $y = 2 + t$, $z = 1 - 3t$

plane: $2x + 2y - 5 = 0$

(b) line: $x = 1 - 2t$, $y = t$, $z = -t$

plane: $6x - 3y + 3z = 1$

a) line: $\vec{d} = \langle -1, 1, -3 \rangle$ $\leftarrow$ directional vector

Plane: $\vec{n} = \langle 2, 2, 0 \rangle$ $\leftarrow$ normal vector ($\perp$ to plane)

$\vec{d} \cdot \vec{n} = -1 \cdot 2 + 1 \cdot 2 + -3 \cdot 0 = 0$ $\quad \therefore$ Parallel

b) line: $\vec{d} = \langle -2, 1, -1 \rangle$

Plane: $\vec{n} = \langle 6, -3, 3 \rangle$

✗ Parallel : $\vec{d} \cdot \vec{n} = -2(6) + 1(-3) + (-1)(3) \neq 0$

✗ Perpendicular: $\vec{d} \neq k\vec{n}$ where $k \in \mathbb{R}$

ie. not scalar multiples of eachother

$\therefore$ Neither