Q1: Find equation of the tangent line to the curve at the given point.

(a) $r = 2 + \sin(3\theta)$, $\theta = \pi/2$

(b) $r = -\sin(2\theta)$, $\theta = \pi/8$

a) $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{dr/d\theta \sin\theta + r\cos\theta}{dr/d\theta \cos\theta - r\sin\theta}$$

$$\frac{dr}{d\theta} = 3\cos(3\theta)$$

$$= \frac{3\cos 3\theta \sin\theta + (2+\sin 3\theta)\cos\theta}{3\cos 3\theta \cos\theta - (2+\sin 3\theta)\sin\theta}$$

$$m = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{3\cos\left(\frac{3\pi}{2}\right)\sin\left(\frac{3\pi}{2}\right) + \left(2+\sin\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)}{3\cos\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right) - \left(2+\sin\frac{3\pi}{2}\right)\sin\left(\frac{3\pi}{2}\right)}$$

$$= \frac{0 + (2-1)(0)}{0 - (2-1)(1)}$$

$$= 0$$

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$

At $\theta = \pi/2$, $r = 2+\sin(3\pi/2) = 2-1 = 1$

$x = 1\cos(\pi/2) = 0$, $y = 1\sin(\pi/2) = 0$

$y - y_0 = m(x - x_0) \Rightarrow y - 1 = 0(x-0) \Rightarrow y = 1$

Q2: Convert the given equation $2r^3\sin(\theta) = 3 - \cos(\theta)$ into a Cartesian equation.

LHS: $2r^3\sin\theta = 2r\,r^2\sin\theta = 2r(r\sin\theta)r = 2r^2 y = 2(x^2+y^2)y$
RHS: $3 - \cos\theta = 3 - x/r$

Hence, $2(x^2+y^2)y = 3 - x/r \Rightarrow 2r(x^2+y^2)y = 3r - x$
$\Rightarrow 2(x^2+y^2)^{3/2}y = 3(x^2+y^2)^{1/2} - x$

Q4: The polar curve $r = 8\cos(\theta) - 5\sin(\theta)$ represents a circle in the xy-plane. Find the Cartesian equation for this circle in standard form, and identify its centre and radius.

$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2$

$r^2 = 8r\cos\theta - 5r\sin\theta \quad \Rightarrow \quad x^2 + y^2 = 8x - 5y$

$\Rightarrow \quad x^2 - 8x + y^2 + 5y = 0$

Now complete the square:

- $x^2 - 8x = x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2$

$= x^2 - 8x + 16 - 16$

$= (x-4)^2 - 16$

- $y^2 + 5y = y^2 + 5y + 25/4 - 25/4$

$= (y + 5/2)^2 - 25/4$

Hence, $(x-4)^2 - 16 + (y + 5/2)^2 - 25/4 = 0$

$\Rightarrow (x-4)^2 + (y + 5/2)^2 = 89/4$

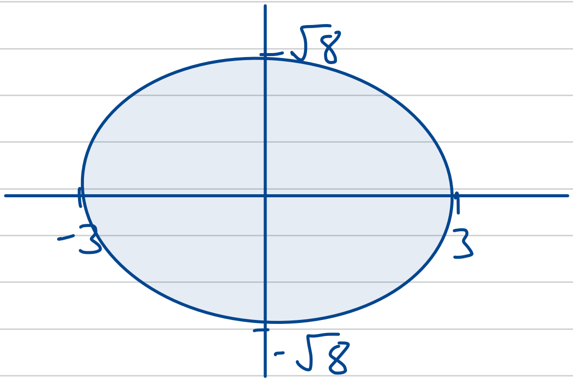
$\therefore$ Center $(4, -5/2)$ Radius : $\sqrt{89}/2$

Q5: Sketch the graph.

(a) $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$ (b) $4x^2 - y^2 = 16$

a) standard form of an ellipse w/

center : $(0,0)$, $a = 3$ along x-axis, $b - \sqrt{8}$ along y axis



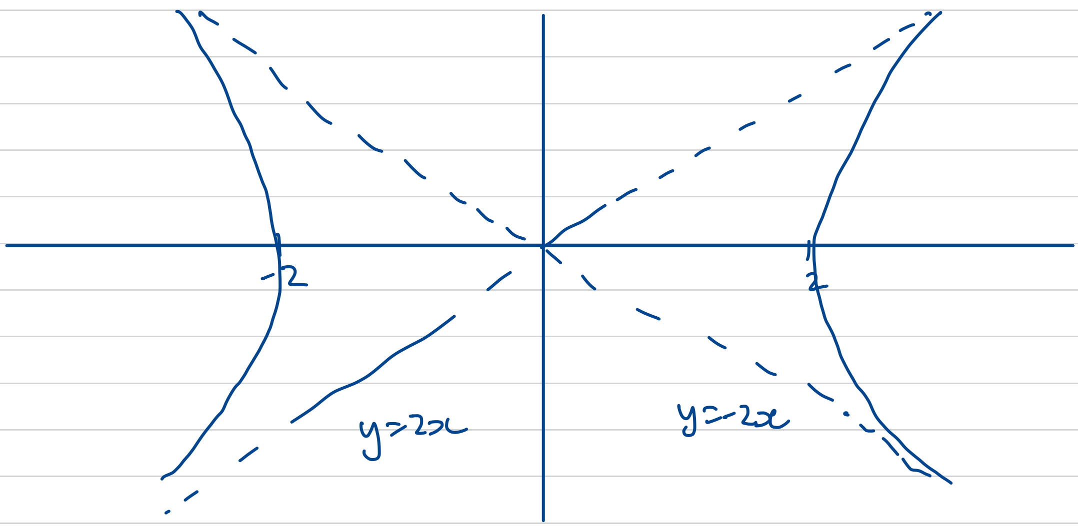
b) $4x^2 - y^2 = 16 \Rightarrow \dfrac{x^2}{4} - \dfrac{y^2}{16} = 1 \qquad (b^2 = 16,\ a^2 = 4)$

Standard form of hyperbola that

- opens left and right (as $x^2/4$ term is positive)
- center $(0,0)$
- vertices at $x = \pm 2$
- asymptotes $y = \pm 2x \quad \left(y = \pm \frac{b}{a}x = \pm \frac{4}{2}x\right)$

Note: The same steps can be used for a hyperbola that opens up/down

i.e. $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$



Q8: Compute $\|\bar{u}\|$, $\|\bar{v}\|$, and unit vector for the given vectors in $\mathbb{R}^3$.

(a) $\bar{u}=15\bar{i}-2\bar{j}+4\bar{k}$, $\bar{v}=\pi\bar{i}+3\bar{j}-\bar{k}$

(b) $\bar{u}=2\bar{k}-\bar{i}$, $\bar{v}=-\bar{k}+\bar{i}$

a) $\|\bar{u}\| = \sqrt{15^2+(-2)^2+4^2} = \sqrt{245}$ $\qquad \|\bar{v}\| = \sqrt{\pi^2+3^2+(-1)^2} = \sqrt{\pi^2+10}$

Unit vector: $\hat{u} = \frac{1}{\sqrt{245}}(15\bar{i}-2\bar{j}+4\bar{k})$

$\hat{v} = \frac{1}{\sqrt{\pi^2+10}}(\pi\bar{i}+3\bar{j}-\bar{k})$

b) $\|\bar{u}\| = \sqrt{(-1)^2+2^2} = \sqrt{5}$ $\qquad \|\bar{v}\| = \sqrt{(-1)^2+1^2} = \sqrt{2}$

Unit vector: $\hat{u} = \frac{1}{\sqrt{5}}(2\bar{k}-\bar{i})$

$\hat{v} = \frac{1}{\sqrt{2}}(-\bar{k}+\bar{i})$