Q1: Evaluate the line integral.

(a) $\int_C x\,ds$

C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$

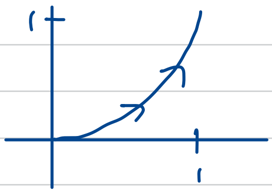
(b) $\int_C yz\cos(x)\,ds$

C: $x = t$, $y = 2\cos(t)$, $z = 3\sin(t)$, $0 \le t \le \pi$

(b) $\int_C xy\,dx + y^2\,dy + yz\,dz$

C is the line segment from $(1,0,-1)$ and $(3,4,2)$

a)



1. Parameterize the curve

let $x = t$, then $y = t^2$ $\quad 0 \le t \le 1$

$\vec{r}(t) = \langle t, t^2 \rangle$

2. Compute $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{1 + (2t)^2}\,dt = \sqrt{1+4t^2}\,dt$

3. Substitute in $\int_C x\,ds = \int_0^1 t\sqrt{1+4t^2}\,dt$

let $u = 1+4t^2 \rightarrow du = 8t\,dt$

$u(0) = 1 \quad u(1) = 5$

$$= \int_1^5 u^{\frac{1}{2}} \frac{1}{8}\,du = \frac{1}{8} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}}\Big|_1^5$$

$$= \frac{1}{12}(5\sqrt{5} - 1)$$

c) 1. Parameterize $\vec{r}(t) = \langle 1+2t, 0+4t, -1+3t \rangle$ on $0 \le t \le 1$

2. Compute $dx = 2\,dt \quad dy = 4\,dt \quad dz = 3\,dt$

Then:

$$xy\,dx = (1+2t)(4t)(2)\,dt = 8t(1+2t)\,dt$$
$$y^2\,dy = (4t)^2(4)\,dt = 64t^2\,dt$$
$$yz\,dz = (4t)(-1+3t)(3)\,dt = 12t(-1+3t)\,dt$$

3. Sub in $\int_C xy\,dx + y^2\,dy + yz\,dz$

$$= \int_0^1 \left[8t(1+2t) + 64t^2 + 12t(-1+3t)\right] dt$$

$$= \int_0^1 (-4t + 116t^2)\, dt$$

$$= \frac{110}{3}$$

Q2: Determine whether or not $\overline{F}$ is a conservative vector field. If it is, find a function f such that $\overline{F} = \nabla f$.

(a) $\overline{F}(x,y) = ye^x\overline{i} + (e^x + e^y)\overline{j}$

(b) $\overline{F}(x,y) = (\ln(y) + y/x)\overline{i} + (\ln(x) + x/y)\overline{j}$

a) $\vec{F}(x,y) = \langle M(x,y), N(x,y)\rangle = \langle ye^x, e^x + e^y\rangle$

$$\frac{\partial M}{\partial y} = e^x = \frac{\partial N}{\partial x}$$

Hence, it is conservative

Next, we want $f(x,y)$ s.t. $\nabla f = \vec{F}$

$$\frac{\partial f}{\partial x} = ye^x \qquad \frac{\partial f}{\partial y} = e^x + e^y$$

$$\int \frac{\partial f}{\partial x}\,dx = y\int e^x\,dx = ye^x + g(y)$$

Then $\frac{\partial f}{\partial y} = e^x + g'(y) \overset{\text{set}}{=} e^x + e^y$

$$\Rightarrow g'(y) = e^y \quad \Rightarrow g(y) = \int e^y\,dy = e^y$$

Hence, such a function $f(x,y) = ye^x + e^y$

Q3: (a) Find a function f such that $\overline{F} = \nabla f$.

(b) Use part (a) to evaluate $\int_C \overline{F} \cdot d\overline{r}$ along the given curve C.

(i) $\overline{F}(x,y) = 2xy\overline{i} + x^2\overline{j}$

$C: \overline{r}(t) = \sin(t)\overline{i} + (1+t)\overline{j} \quad , \quad 0 \le t \le \pi$

(i) $\overline{F}(x,y,z) = e^z\overline{i} + xz\overline{j} + (x+y)\overline{k}$

$C: \overline{r}(t) = t^2\overline{i} + t^3\overline{j} - t\overline{k} \quad , \quad 0 \le t \le 1$

(i) a. Let $\vec{F} = \nabla f = \langle \partial f/\partial x , \partial f/\partial y \rangle$

$= \langle 2xy , x^2 \rangle$

Then $f(x,y) = \int 2xy\, dx = x^2 y + g(y)$

Now, $\frac{\partial f}{\partial y} = x^2 + g'(y) \overset{\text{set}}{=} x^2$

$\Rightarrow g'(y) = 0 \Rightarrow g'(y) = k$ for $k \in \mathbb{R}$

So Potential Function is $f(x,y) = x^2 y$

b. $\vec{r}(t) = \langle \sin t, 1+t \rangle \quad 0 \le t \le \pi$

As $\vec{F}$ is conservative check! and potential function $f(x,y) = x^2 y$,

$\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(\pi)) - f(\vec{r}(0))$

$\vec{r}(\pi) = \langle \sin \pi, 1+\pi \rangle = \langle 0, 1+\pi \rangle$

$\vec{r}(0) = \langle \sin 0, 1+0 \rangle = \langle 0, 1 \rangle$

Then $\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(\pi)) - f(\vec{r}(0))$

$= f(\langle 0, 1+\pi \rangle) - f(\langle 0, 1 \rangle)$

$= 0^2 (1+\pi)^2 - 0^2 (1)^2$

$= 0$