Q1: Calculate the iterated integral.

(a) $\displaystyle\int_{\pi/2}^{\pi}\int_{0}^{2\sin(\theta)} r\,dr\,d\theta$

(b) $\displaystyle\iint_R \frac{y^2}{x^2+y^2}\,dA$, where R is the region that lies between the circles $x^2+y^2=a$ and $x^2+y^2=b$ where $0<a\le b$.

(c) $\displaystyle\iint_D \cos\sqrt{x^2+y^2}\,dA$, where D is the disk with centre being the origin and radius 2.

(d) $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

a) $$\int_{\pi/2}^{\pi}\underbrace{\int_0^{2\sin\theta} r\,dr}\,d\theta$$

$$\int_{\pi/2}^{\pi} \frac{1}{2}r^2\Big|_0^{2\sin\theta}\,d\theta = \int_{\pi/2}^{\pi} \frac{1}{\cancel{2}}\cancel{4}\sin^2\theta\,d\theta$$

$$= \int_{\pi/2}^{\pi} 1-\cos(2\theta)\,d\theta$$

$$= \left[\theta - \frac{1}{2}\sin(2\theta)\right]\Big|_{\pi/2}^{\pi}$$

$$= (\pi - \pi/2) - \frac{1}{2}(0-0) = \pi/2$$

c) Recall: $x^2+y^2=r^2 \Rightarrow r=\sqrt{x^2+y^2}$ $\quad dA = r\,dr\,d\theta$

$$\iint_D \cos(\sqrt{x^2+y^2})\,dA = \int_0^{2\pi}\underbrace{\int_0^{2}\cos r\; r\,dr}\,d\theta$$

$u = r \Rightarrow du = dr$
$dv = \cos r\,dr \Rightarrow v = \sin r$

$$\int \cos r\; r\,dr = r\sin r - \int \sin r\,dr = r\sin r + \cos r$$

From θ to 2: $\left(2\sin(2)+\cos(2)\right)-(0+1)$

$= 2\sin(2)+\cos(2)-1$

$$= \int_0^{2\pi} 2\sin(2)+\cos(2)-1\, d\theta$$

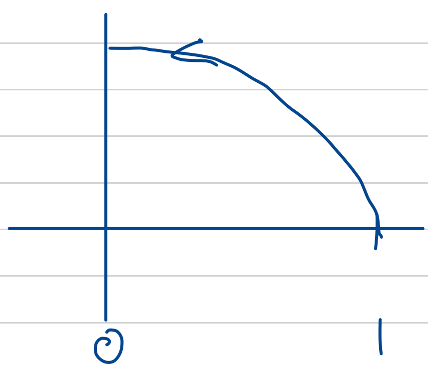
$$= (2\sin 2+\cos 2-1)\,2\pi$$

Q2: Express the iterated integral as a double integral in polar coordinates. DO NOT EVALUATE.

(a) $\displaystyle\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x}\left(\frac{1}{x^2+y^2}\right)dy\,dx$

(b) $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}(x^2+y^2)\,dy\,dx$

b)



$x^2+y^2=r^2$, $dy\,dx = r\,dr\,d\theta$

$r \in (0, 1)$

$\theta = (0, \pi/2)$

$$\int_0^{\pi/2}\int_0^1 r^2 \cdot r\,dr\,d\theta.$$

Q3: Compute the Jacobian for each transformation.

(a) $x = 5u - 3v^2 \qquad y = u^2 - 6v^2$

(b) $x = u^2 v^3 \qquad y = -\sqrt{v} - 4$

a) $J = \det \begin{pmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{pmatrix} = \det \begin{pmatrix} 5 & -6v \\ 2u & -12v \end{pmatrix}$

$= -60v + 12uv$

b) $J = \det \begin{pmatrix} 2uv^3 & 3u^2v^2 \\ 0 & -\frac{1}{2\sqrt{v}} \end{pmatrix} = -u \cdot v^{\frac{5}{6}}$