Q1: Determine which of the following statements are **FALSE**.

I. If f and g are continuous on $[a,b]$, then $\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. If f has an absolute minimum value at c, then $f'(c)=0$.

III. If $f'(x)<0$ for $1<x<6$, then f is decreasing on $(1,6)$.

IV. All continuous functions have derivatives.

V. If f is continuous on $[a,b]$, then $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = f(x)$

I. let $f(x), g(x) = x$ on $[0,1]$.

$$\int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} \neq$$

$$\left(\int_0^1 x\, dx\right)^2 = \left(\frac{x^2}{2}\Big|_0^1\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

II. Consider $f(x) = |x|$ has ab. min at $c=0$. But $f'(0)$ DNE

IV. Consider $f(x) = |x|$ is not differentiable at $x=0$

V. let $f(x) = x$ on $[0,1]$, then

$$\frac{d}{dx}\int_0^1 x\,dx = \frac{d}{dx}\frac{x^2}{2}\Big|_0^1 = \frac{d}{dx}\left(\frac{1}{2}-0\right) = 0 \neq f(x)$$

Q2: Only one of the following statements is always true for any case. Determine which one is true. (Assume that $f(x)$ and $g(x)$ are continuous.)

A. $\int_1^2 f(x^3)dx = \int_1^8 \frac{f(u)}{3u^{2/3}}du$

B. $\int_0^1 [f(x)+g(x)]dx = -\int_1^0 f(x)dx + \int_0^1 g(x)dx = 0$

C. If $\int_a^b f(x)dx \geq 0$, then $f(x) \geq 0$ on $[a,b]$.

D. $\int_{-3}^{-2} f(x)dx = -\int_2^3 f(x)dx$

E. If f is continuous on $[a,b]$, then $\int_a^b xf(x)dx = x\int_a^b f(x)dx$.

A. let $u = x^3 \Rightarrow du = 3x^2 dx$

$x = u^{1/3} \Rightarrow dx = \frac{1}{3x^2}du = \frac{1}{3u^{2/3}}du$

$u(1) = 1$, $u(2) = 2^3 = 8$

$$\int_1^2 f(x^3)dx = \int_1^8 \frac{f(u)}{3u^{2/3}}du$$

Q3: If f is a differentiable function on an open interval (a,b) and x is a real number in (a,b), which of the following statements is **NOT** always true ?

A. $\int_a^x f'(t)dt = f(x) - f(a)$ FTC

B. $\frac{d}{dx}\left(\int_a^b f(t)dt\right) = 0$ See Q1 V

C. $\int_a^x f(t)dt = f'(x) - f'(a)$ See below

D. $\int f'(x)dx = f(x) + C$

E. If $F'(x) = f(x)$, then $\int_a^x f(t)dt = F(x) - F(a)$

F. $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$

FTC I, II

C. let $f(t) = t^2$ on $(0, x)$.

$$\int_0^x t^2 dt = \frac{t^3}{3}\Big|_0^x = \frac{x^3}{3} \neq$$

$$f'(x) - f'(0) = 2x$$

Q1: Let $y=\sqrt{x+f(x^2-1)}$. Given that $f(8)=0$ and $f'(8)=3$, find $\frac{dy}{dx}$ when $x=3$.

$$\frac{dy}{dx}=\frac{1}{2}\left(x+f(x^2-1)\right)^{-\frac{1}{2}}\left(1+f'(x^2-1)\cdot 2x\right)$$

$$\left.\frac{dy}{dx}\right|_{x=3}=\frac{1}{2}\left(3+f(8)\right)^{-\frac{1}{2}}\left(1+f'(8)\cdot 2(3)\right)$$

$$=\frac{19}{2\sqrt{3}}$$

Q3: Given that $f''(u)=\dfrac{2u^2-4\sqrt{u}}{u}$ with the initial conditions $f'(0)=2$ and $f(0)=-1$, find $f(u)$.

$$f''(u)=2u-4u^{-\frac{1}{2}}$$

$$f'(u)=\int f''(u)\,du=2\cdot\frac{u^2}{2}-4\,\frac{u^{1/2}}{1/2}+C_1$$

$$=u^2-8u^{1/2}+C_1$$

$$f'(0)=2 \Rightarrow 0^2-8(0)^{1/2}+C_1=2 \Rightarrow C_1=2$$

$$\therefore f'(u)=u^2-8u^{1/2}+2$$

$$f(u)=\int f'(u)\,du=\int u^2-8u^{1/2}+2\,du$$

$$=\frac{u^3}{3}-8\,\frac{u^{3/2}}{3/2}+2u+C_2$$

$$=\frac{u^3}{3}-\frac{16}{3}u^{3/2}+2u+C_2$$

$$f(0)=-1 \Rightarrow 0-0+0+C_2=-1 \Rightarrow C_2=-1$$

$$\therefore f(u)=u^3/3-16/3\,u^{3/2}+2u-1$$

Q5: Suppose that f and h are continuous functions and that $\int_1^7 f(x)dx = -1$, $\int_1^9 f(x)dx = 5$, and $\int_7^9 2h(x)dx = 4$. Find $\int_7^9 \left[-f(x)+3h(x)\right]dx$.

① $\int_7^9 f(x)\,dx$

$$\int_1^9 f(x)\,dx = \int_1^7 f(x)\,dx + \int_7^9 f(x)\,dx$$

$$\Rightarrow \quad 5 = -1 + \int_7^9 f(x)\,dx$$

$$\Rightarrow \int_7^9 f(x)\,dx = 6$$

② $\int_7^9 h(x)\,dx$

$$\int_7^9 2h(x)\,dx = 4 \Rightarrow \int_7^9 h(x)\,dx = 2$$

③ $\int_7^9 [-f(x) + 3h(x)]\,dx = \int_7^9 -f(x)\,dx + \int_7^9 3h(x)\,dx$

$$= -\int_7^9 f(x)\,dx + 3\int_7^9 h(x)\,dx$$

$$= -6 + 3(2)$$

$$= 0$$